7th
Sensometrics
Meeting
July 30, 2004

TIME INTENSITY curves:

statistical treatment

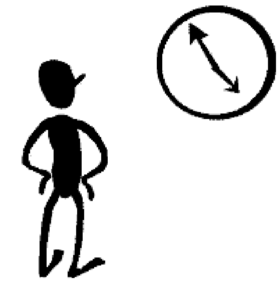
Stéphanie LEDAUPHIN, Evelyne VIGNEAU
El Mostafa QANNARI

OVERVIEW

- **Time intensity curves**
  - **Description**
  - **A high variability between assessors meaning difficulties for the analysis**
- Statistical processing of time intensity data
  - Pretreatment and average among repetitions
  - The ‘ skeleton ’, a tool to describe the ‘signature’
  - Non linear transformation of time in order to reduce the ‘ signature ’ effect
  - Characterization of the products

TIME INTENSITY CURVES :
Description

• Different evolution of the intensity overtime from one product to another

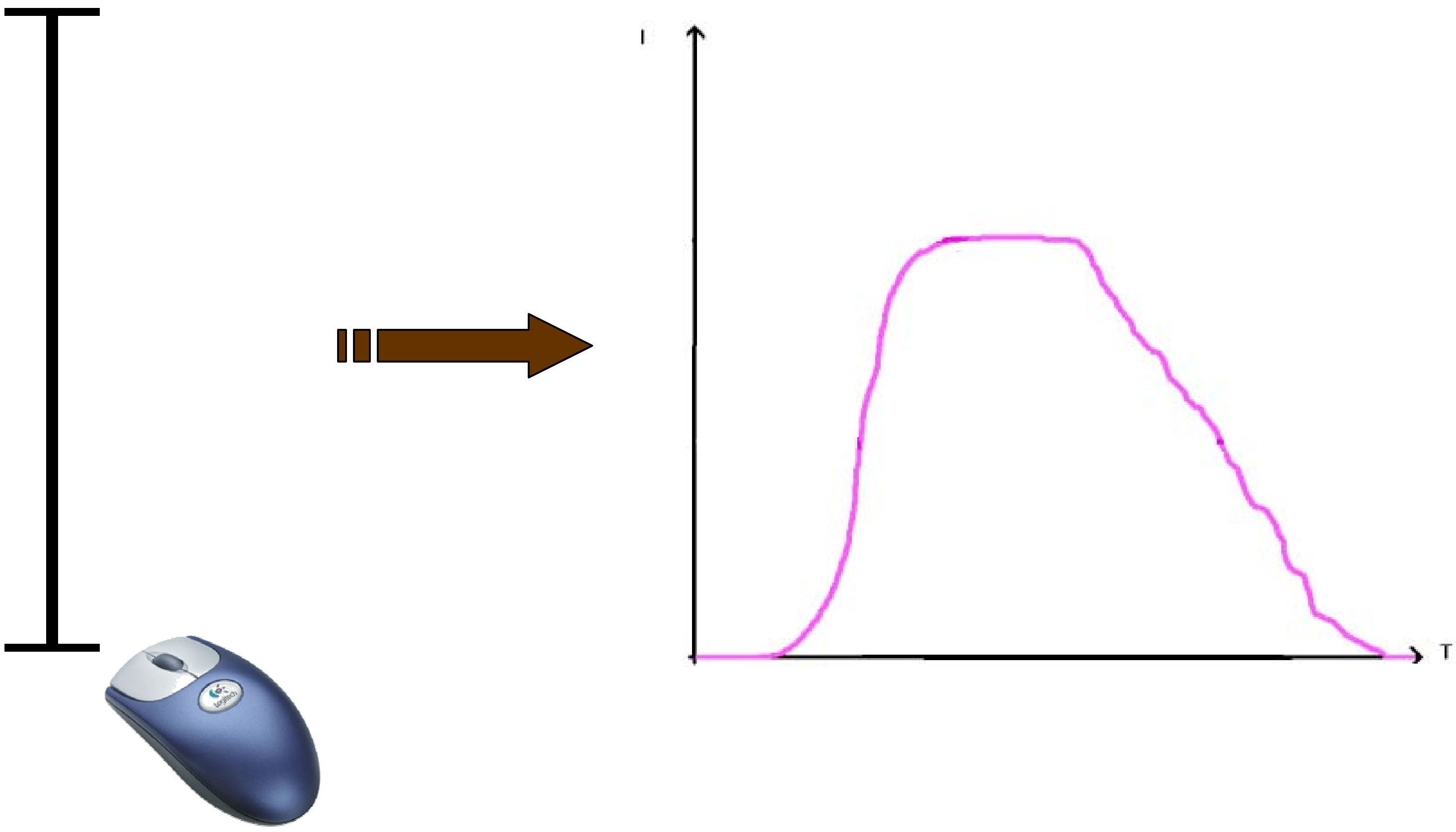


⇨ More complete sensory characterization

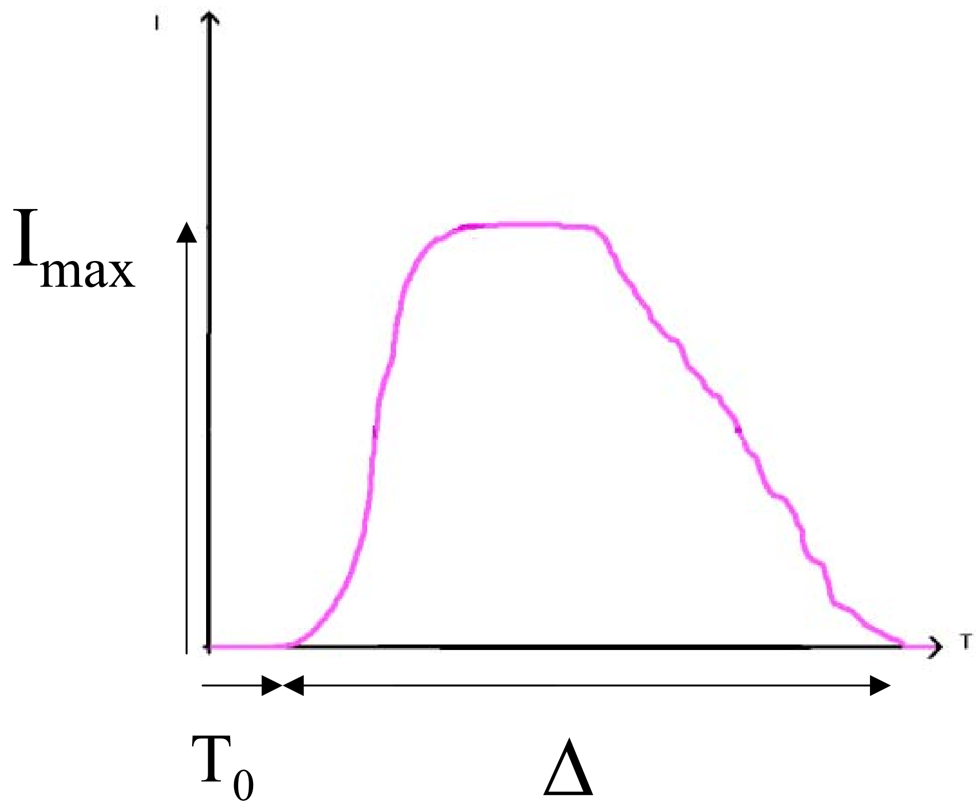
7th Sensometrics Meeting
July 30, 2004

TIME INTENSITY CURVES :

Description



TIME INTENSITY CURVES :
Description

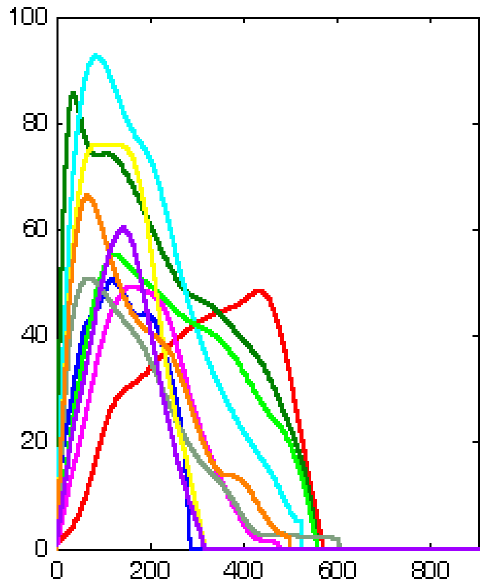


7th Sensometrics Meeting
July 30, 2004

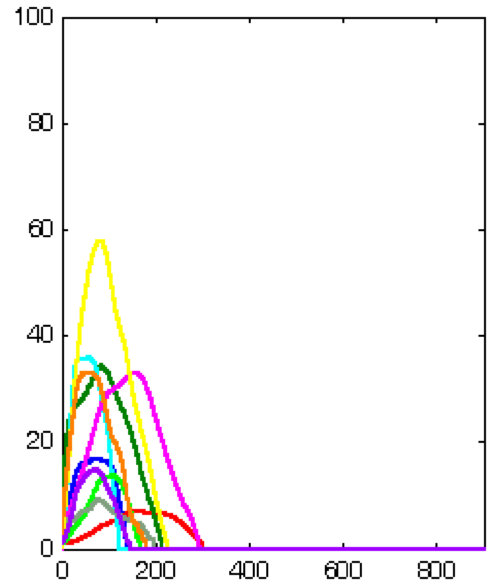
TIME INTENSITY CURVES :

Difficulties to Analyze the Curves...

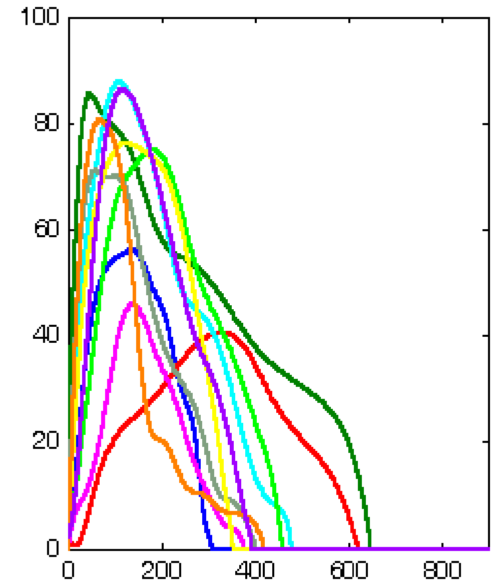
all assessors
product 2



all assessors
product 3



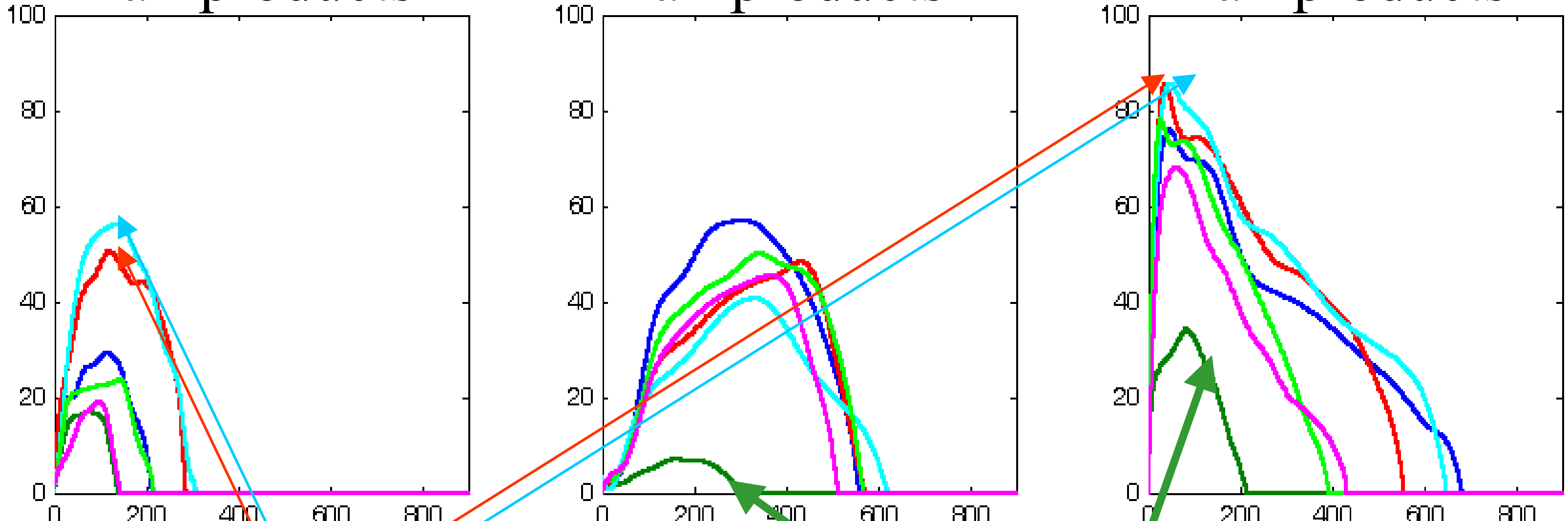
all assessors
product 4



TIME INTENSITY CURVES :

…because of a high variability between assessors

assessor 1 all products | assessor 2 all products | assessor 3 all products



Products 2 and 4 have a large perceived intensity with a fast increase.

The product 3 has a small perceived intensity with a slow increase and a small duration

SAMPLES

Sweetness of Six chocolate drinks

- 10 experienced assessors
- 3 repetitions
- The time of recording = 90 seconds

(with a point of measurement all the tenths of a second).

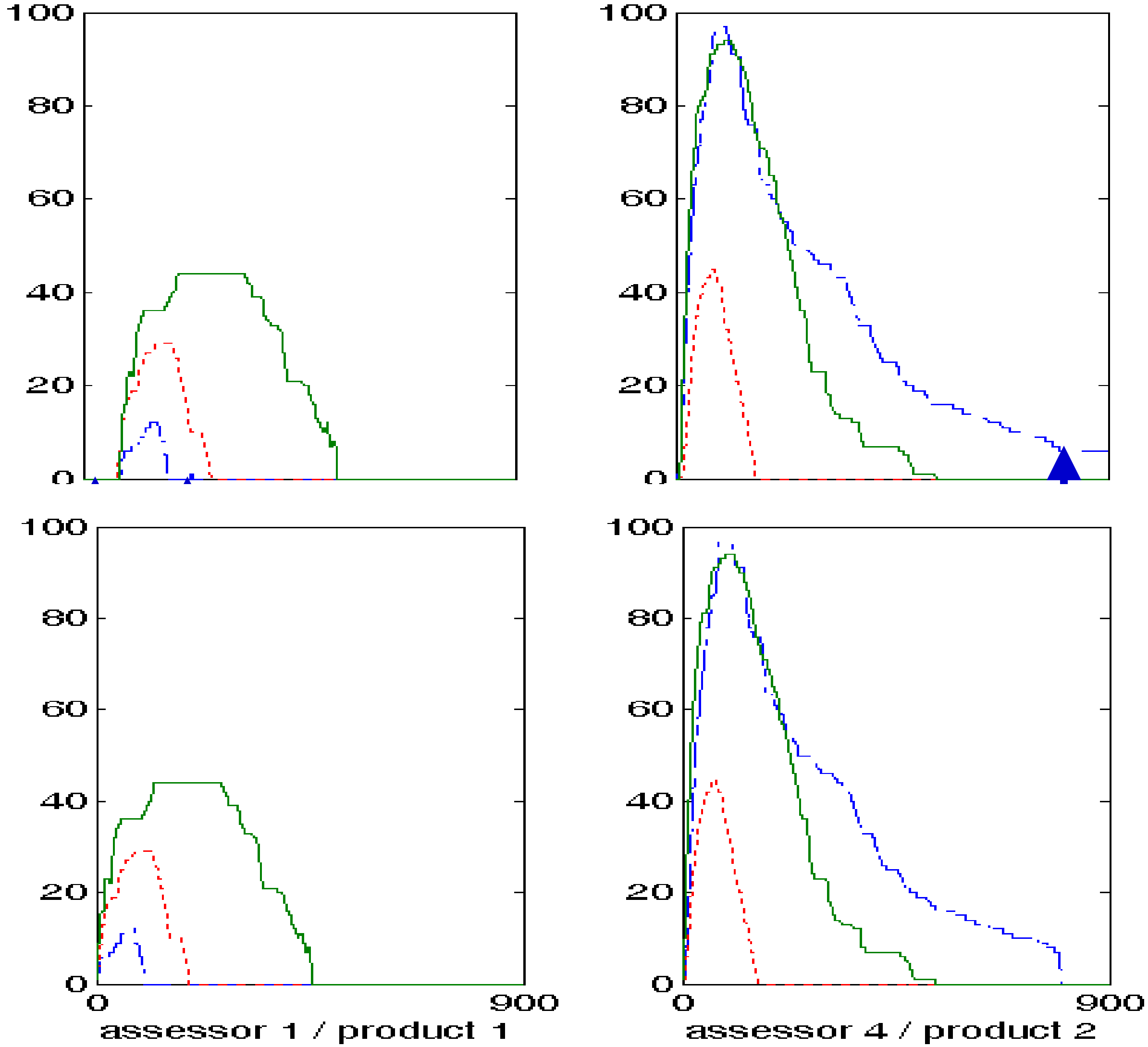
Campden & Chorleywood Food Research Association, England

OVERVIEW

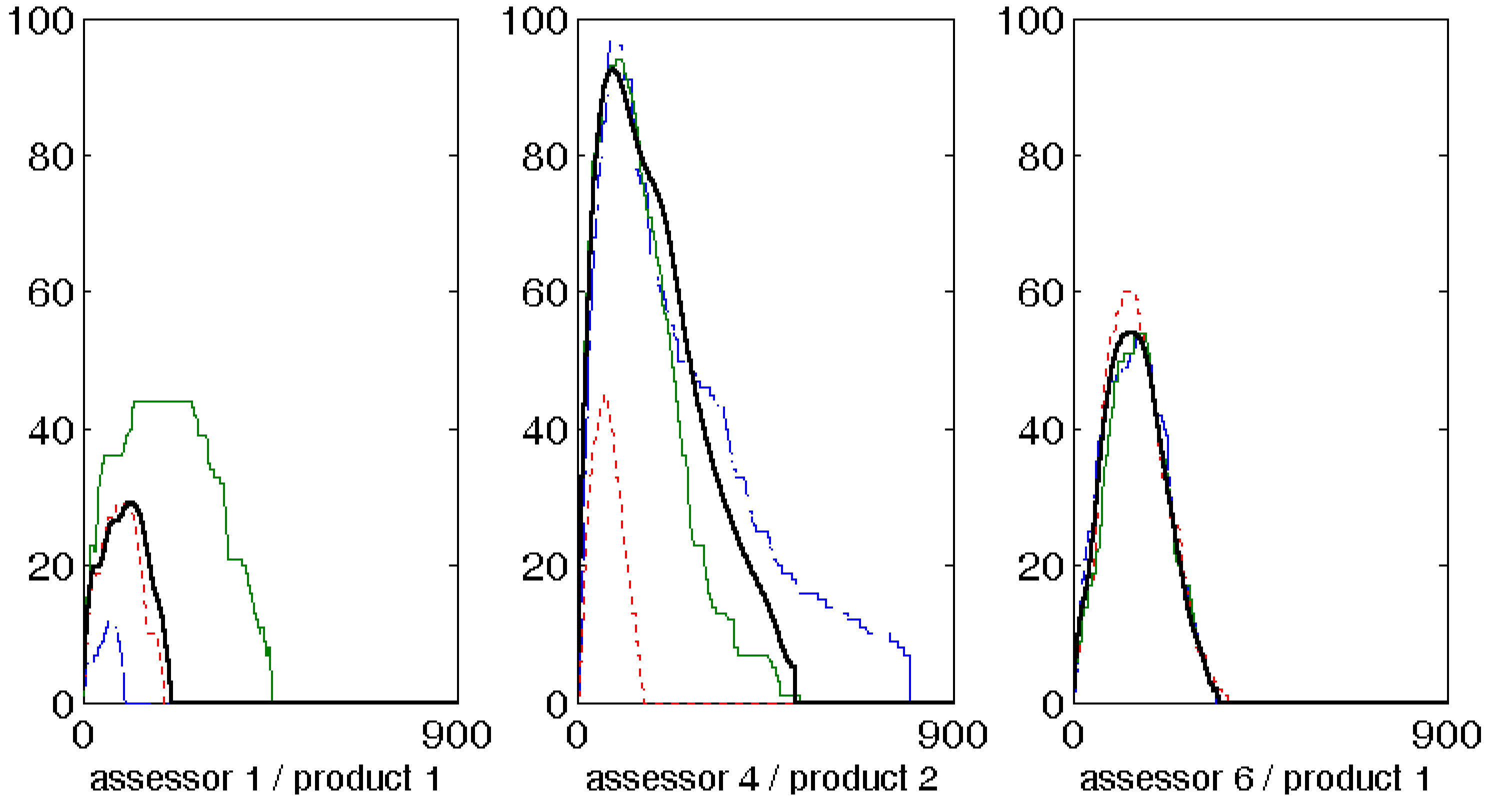
- Time intensity curves
  - Description
  - A high variability between assessors meaning difficulties for the analysis.
- **Statistical processing of time intensity data**
  - Pretreatment and average among repetitions
  - The ‘ skeleton ’, a tool to describe the ‘signature’
  - Non linear transformation of time in order to reduce the ‘ signature ’ effect
  - Characterization of the products

7th Sensometrics Meeting
July 30, 2004

PRETREATMENT



WEIGHTED AVERAGE OVER REPETITIONS



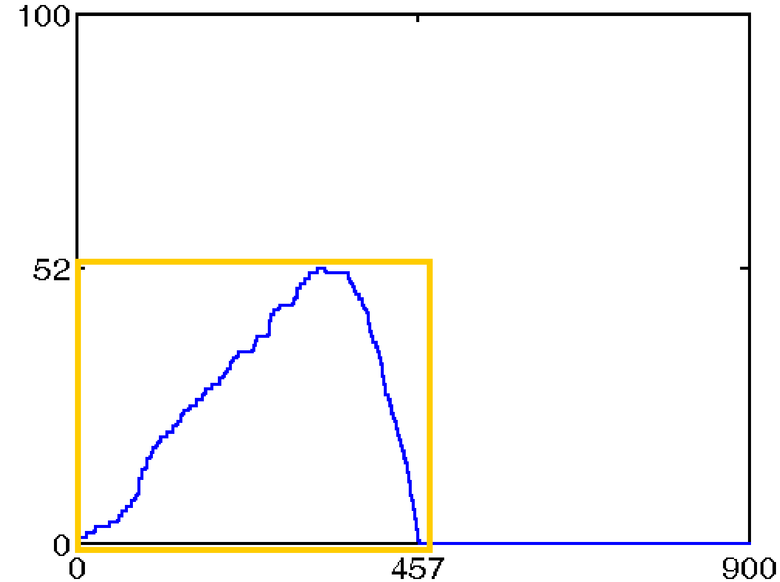
THE ‘ SKELETON ’

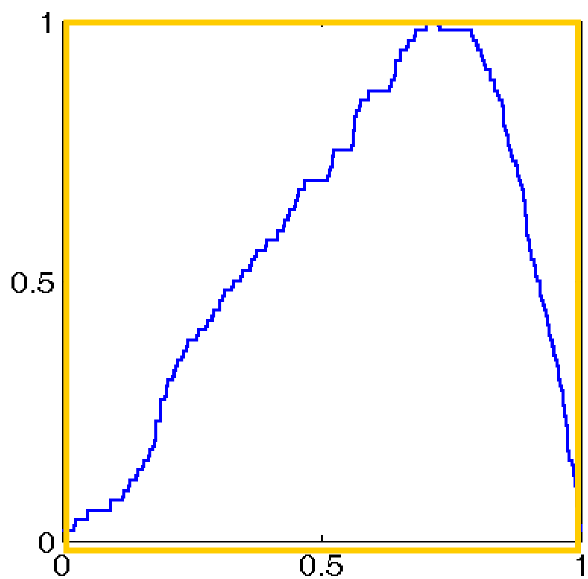
a Tool to Describe Shape of Curves

- Select I_{max} and Δ

  $I_{max} = 52 \quad \Delta = 457$

- Keep only the shape: $[0,1]x[0,1]$





skeleton

THE ‘ SKELETON ’

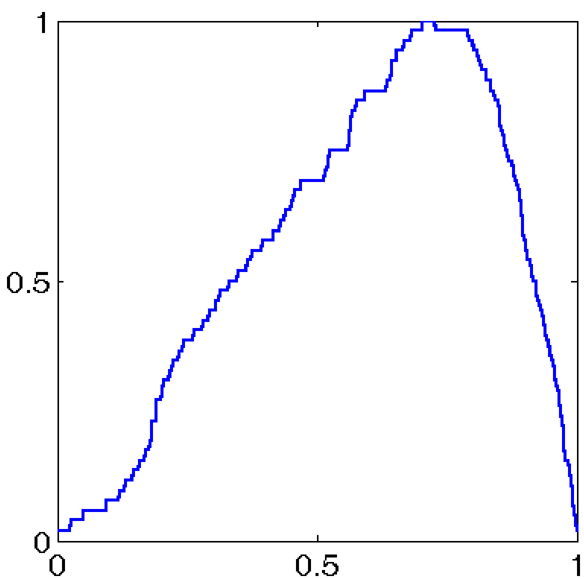
a Tool to Describe Shape of Curves

- Select I_{max} and Δ

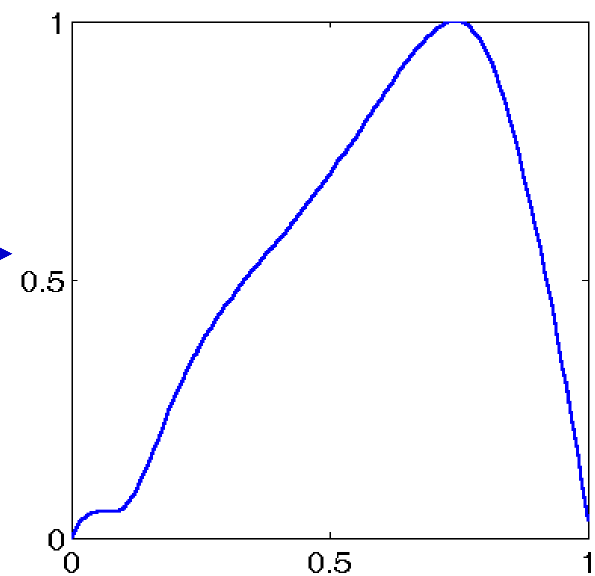
  $I_{max} = 52 \quad \Delta = 457$

- Keep only the shape: [0,1]x[0,1]

- Project in a basis of spline functions



skeleton



$$c = \begin{bmatrix} 0.0 \\ 0.1 \\ 0.0 \\ 0.3 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 1.0 \\ 1.0 \\ 0.6 \\ 0.3 \\ 0.0 \end{bmatrix}$$

THE ‘ SKELETON ’

a Tool to Describe Shape of Curves

Formally, the standardization is :

skeleton

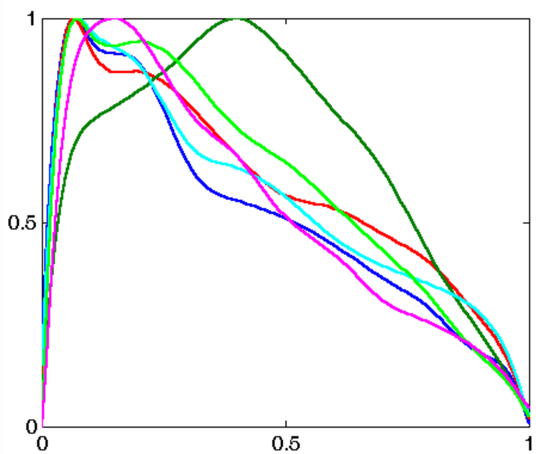
$$y_{ij}(t) = \mathrm{Imax}_{ij}\, Z_{ij}\left(\frac{t}{\Delta_{ij}}\right) + \varepsilon_{ij}(t)$$

And the decomposition on a B-splines basis:

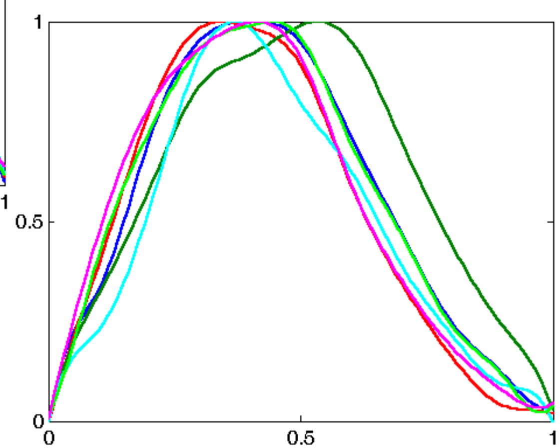
$$Z(t^{*}) = c_1\Phi_1(t^{*}) + c_2\Phi_2(t^{*}) + \ldots + c_S\Phi_S(t^{*})$$

THE ‘ SIGNATURE ’

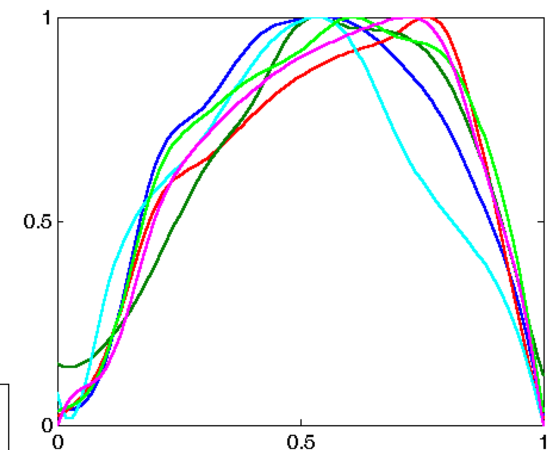
The skeleton reflects the signature of each assessor



assessor 6
all products



assessor 3
all products



assessor 2
all products

OBJECTIVES

⇨ To remove the variability between assessors with the aim of better discriminating the products.

METHOD

Assessor effect for the parameters Imax and Δ:

$Imax_{ij}$ for each product *i* and assessor *j*

$Imax_{.j}$: average for assessor *j*

$Imax_{..}$: global average

⇨ $Imax_{ij}' = Imax_{ij} + (Imax_{..} - Imax_{.j})$

⇨ $\Delta_{ij}' = \Delta_{ij} + (\Delta_{..} - \Delta_{.j})$

METHOD

Assessor effect for the ' skeleton ':

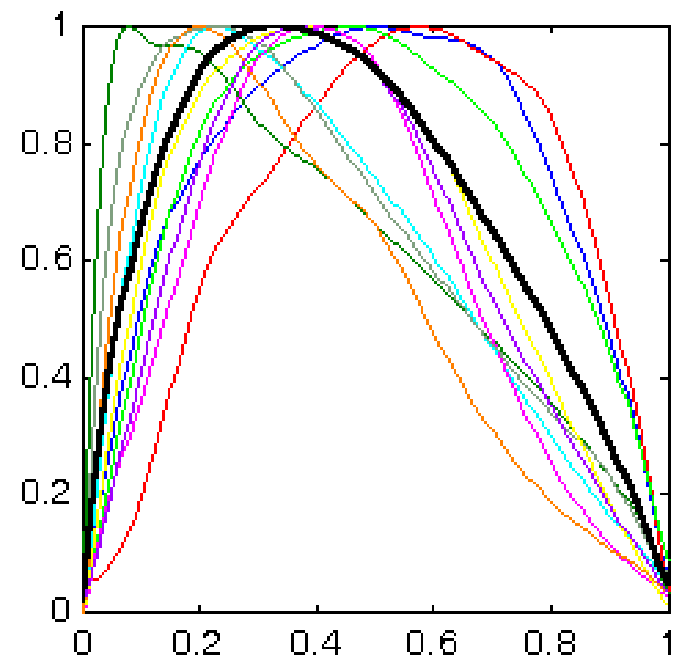
Z_{ij} for each product *i* and assessor *j*

$Z_{.j}$: average for assessor *j*

$Z_{..}$: global average

$Z_{ij}' = ?$

$Z_{ij}'(t) = Z_{ij}(u_j(t))$



References

« Landmarks based registration of curves »

Kneip and Gasser (1992)

« Dynamic time warping »

Wang and Gasser (1997), Ramsay and Silverman (1997)

Formalization of DTW

For each assessor j, we have a function $Z_{.j}$ representing his or her signature (average skeleton). We are looking for the transformation u_j so that:

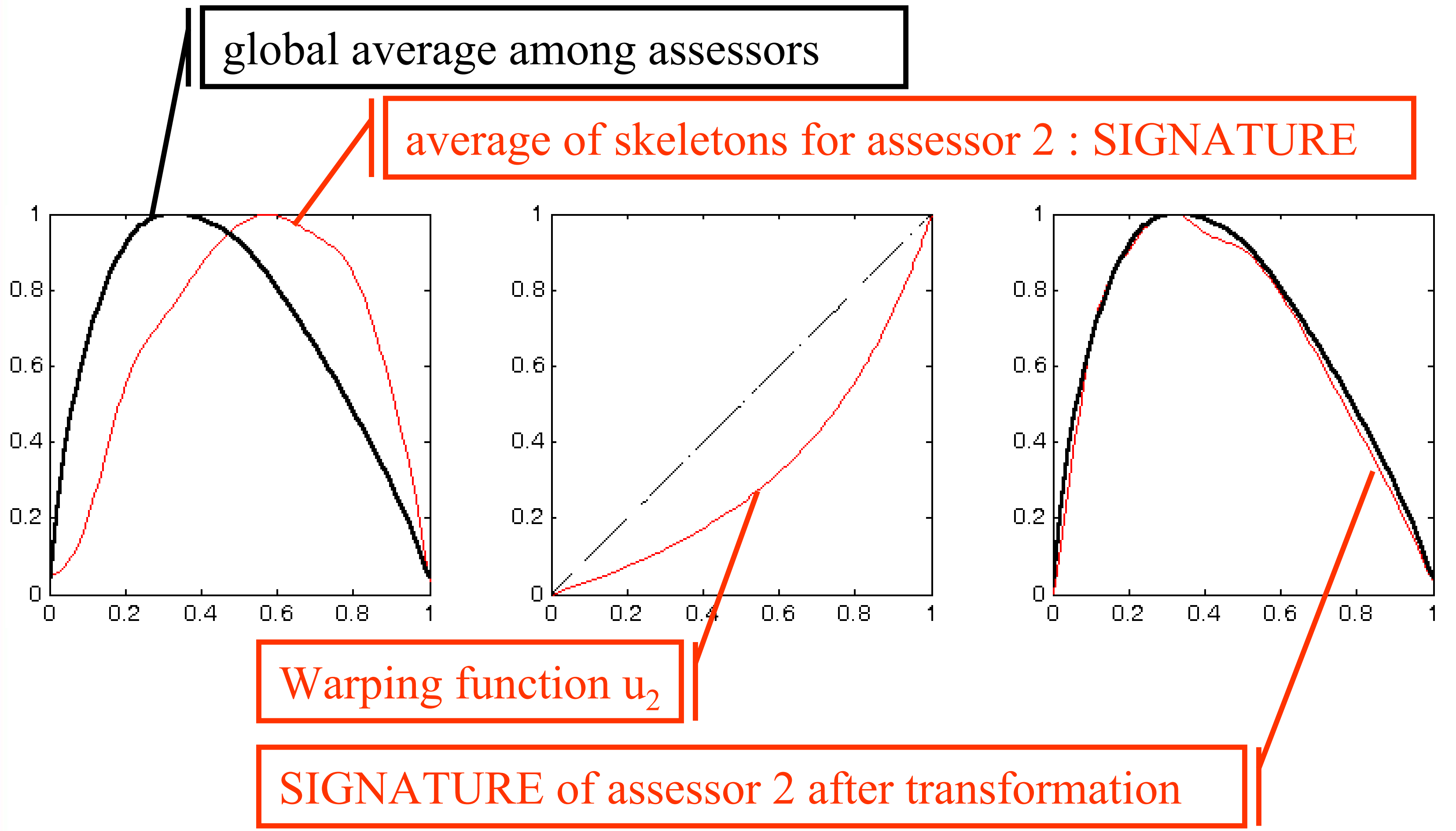
$$Z_{.j}\left(u_j(t)\right) = Z_{..}(t)$$

where Z.. is the global average skeleton.

The principle of DTW: $$\underset{u_j,\, j=1..m}{\mathrm{Min}} \sum_{j=1}^{m} \left\| Z_{.j}\left(u_j(t)\right) - Z_{..}(t) \right\|^2$$

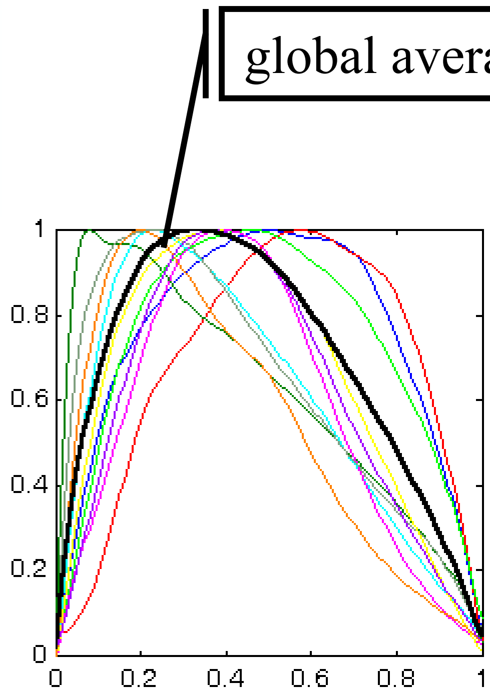
7th Sensometrics Meeting
July 30, 2004

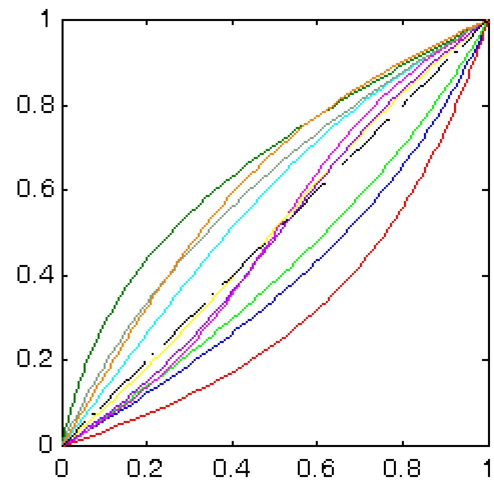
RESULTS : assessor 2



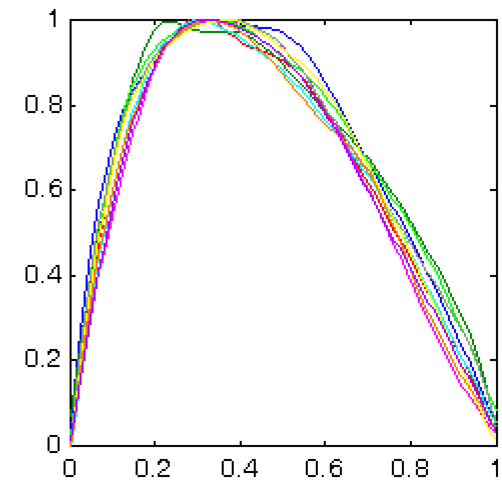
7th Sensometrics Meeting July 30, 2004

RESULTS : all assessors



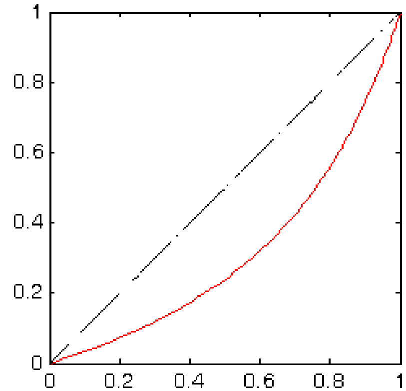


Warping functions u_j

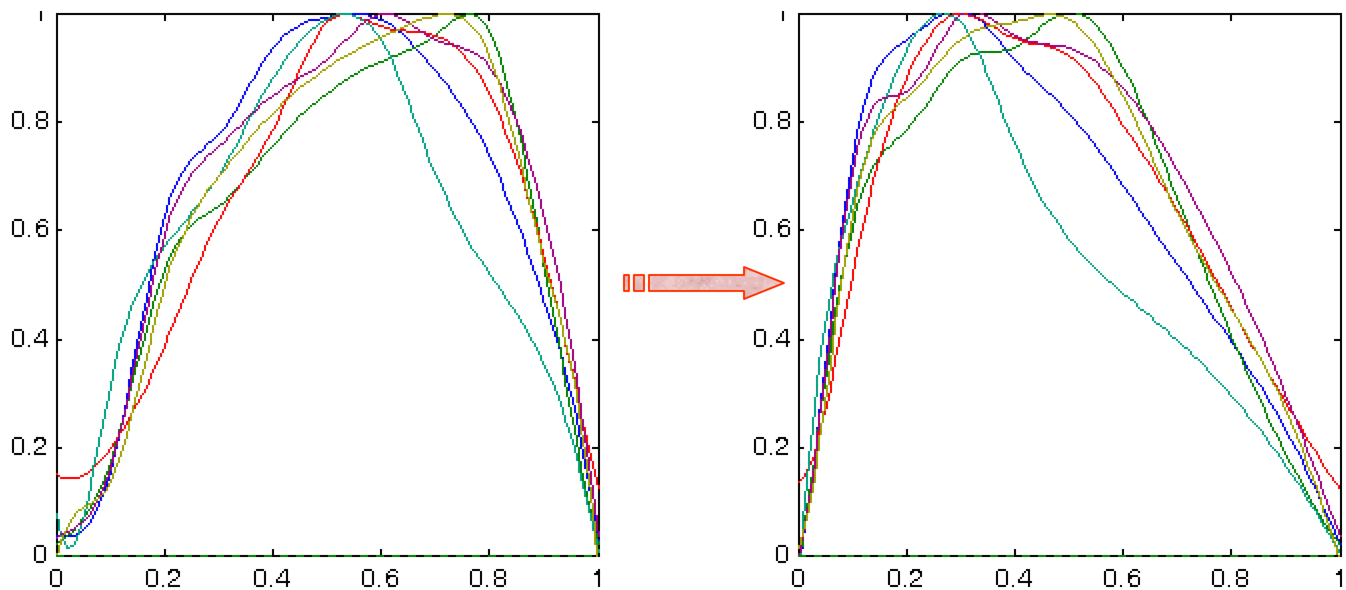


average of skeletons after transformation

RESULTS : assessor 2

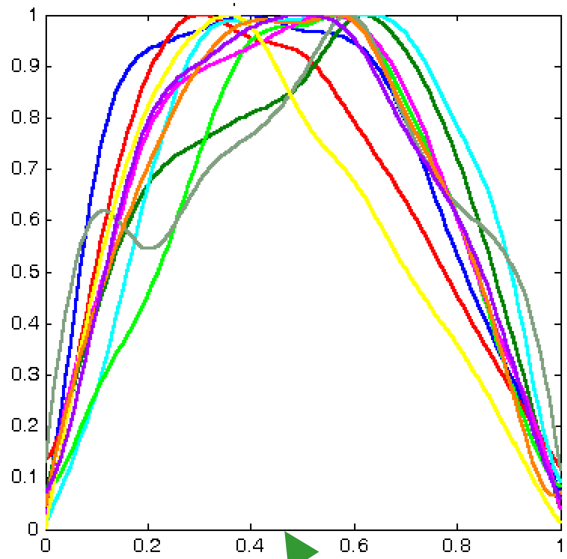


Warping function u_2 is applied to all skeletons associated with this assessor

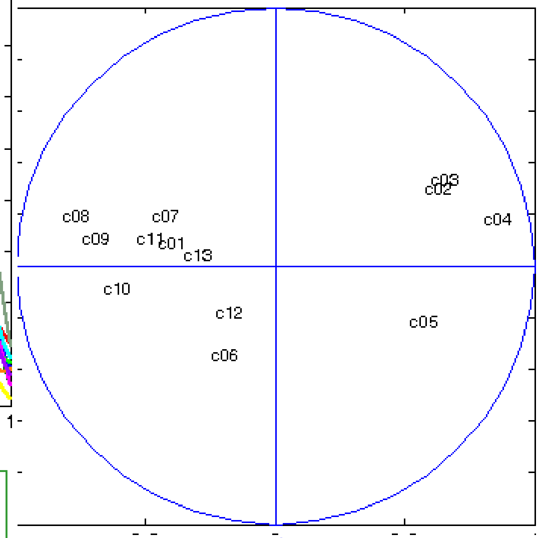


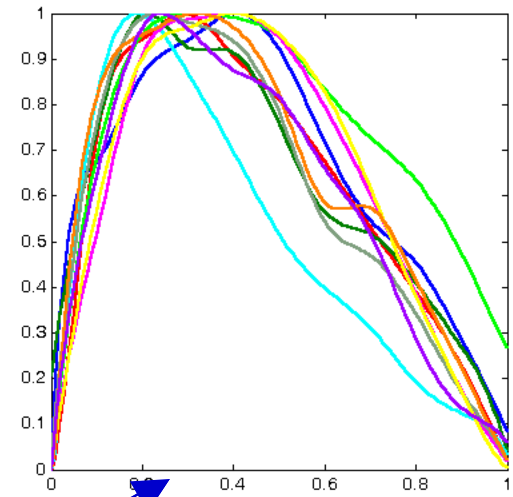
7th Sensometrics Meeting July 30, 2004

7th Sensometrics Meeting
July 30, 2004



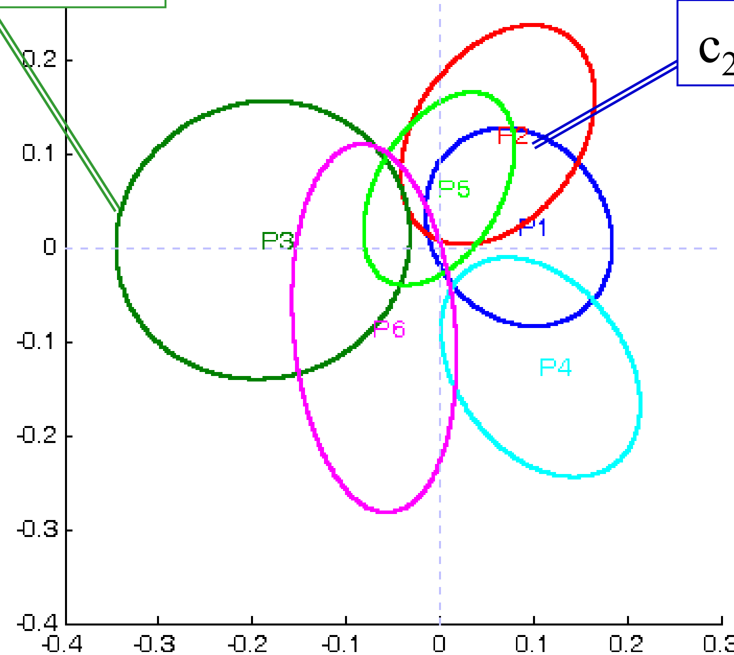
c_2, c_3 , c_4 and c_5 small

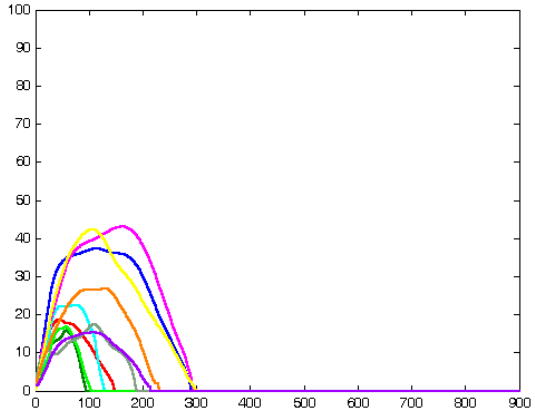




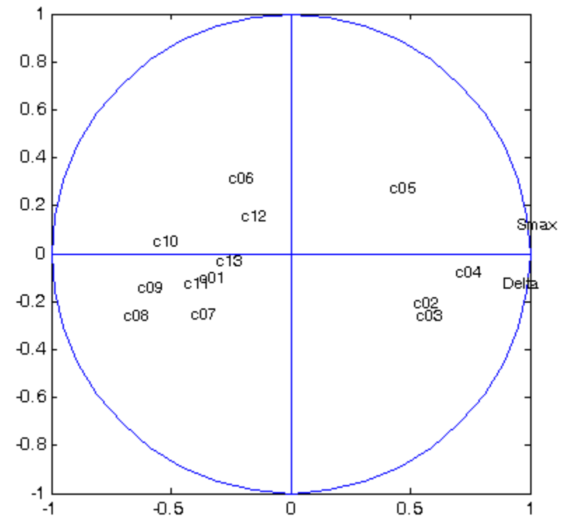
c_2, c_3 , c_4 and c_5 high

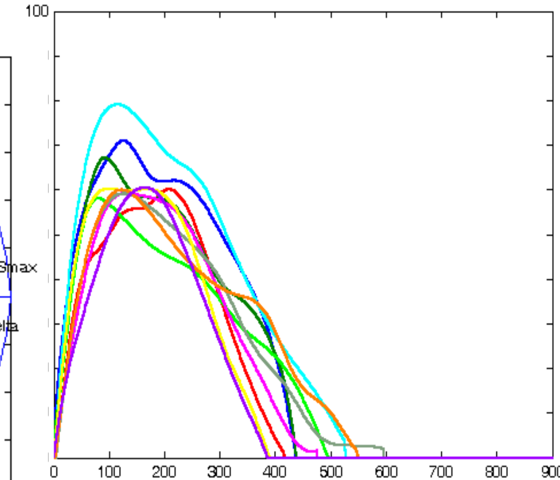
Fisher 's Discriminant Analysis



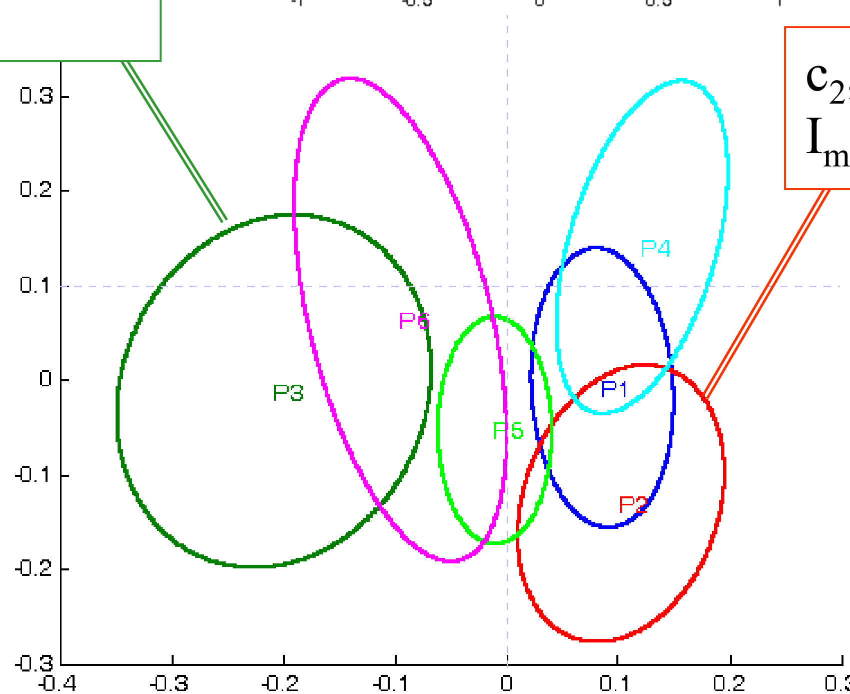


c_2, c_3 , c_4, c_5, Δ and I_{max} small





c_2, c_3 , c_4, Δ and I_{max} high



CONCLUSION

- reduce assessor effect
  - variability in I_{max}
  - variability in Δ
  - variability in the shape of curves (skeleton)
- improve the characterization of the products
- compute an estimation of prototype curve by product

References

DIJKSTERHUIS G.B. & EILERS P. (1997). Modelling time-intensity curves using prototype curves.

KNEIP A. & GASSER T. (1992). Statistical tools to analyze data representing a sample of curves.

RAMSAY J.O. (1998). Curve registration.

VIGNEAU E., LEDAUPHIN S. & CAUSEUR D. (2004). Approche fonctionnelle de l'analyse des signaux temps intensité.

WANG K. & GASSER T. (1997). Alignment of curves by dynamic time warping.

- **Time intensity curves**

  description (d.6), A great variability between assessors (d.7)

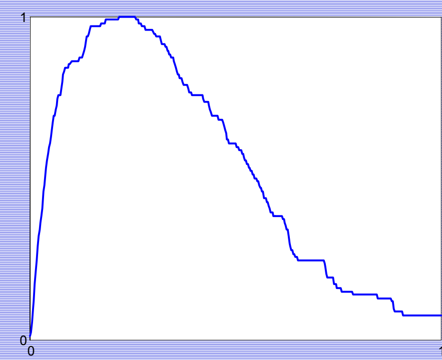
- **Statistical processing of temporal data**

  pretreatment (d.11), ‘skeleton’ (d.13), ‘signature’ (d.16), non linear transformation of times (d.20), dtw (d.23) characterization of the products (d.25)

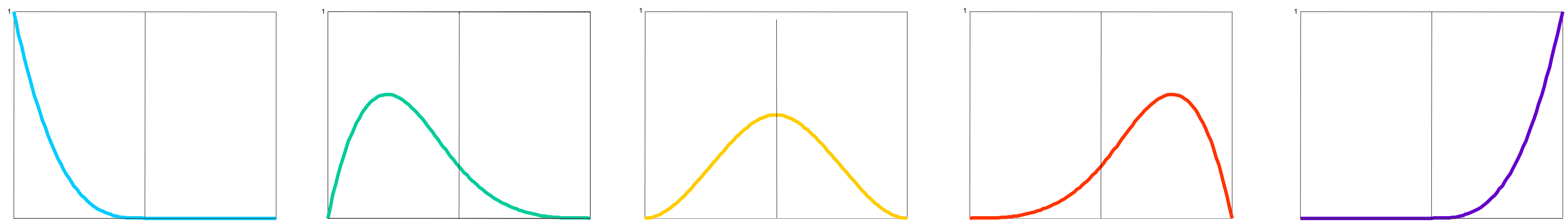
Questions...

- Spline basis
- Compromise among repetitions
- Repeatability
- Analysis of variance
- Fisher ’s discriminant analysis

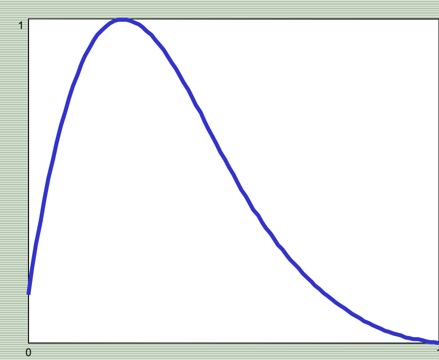
Spline Basis



B-splines : degree = 3, 1 knot ➔ 5 basis functions



0.21	*1.53*	*0.31*	*0.09*	*0.09*



7th Sensometrics Meeting July 30, 2004

Compromise Among Repetitions

Weighted average of B-splines coefficients by taking account of a matrix of similarity

Weighted average of the parameters I_{max} and Δ

Computation of the compromise

Matrix of Similatity coefficients of the B-splines basis

$$m_{kh}^{1/2} = 2 \star \frac{\left\langle c_{ijk}, c_{ijh} \right\rangle}{\left\| c_{ijk} \right\|^2 + \left\| c_{ijh} \right\|^2}$$

For each assessor j and product i:

m_{kh} are indices of similarity between repetitions k and h

Weigth for each repetitions

For each assessor j and product i, for a parameter X (I_{max}, Δ), the weight of repetition k is given by:

$$p_{ijk}^{(X)} = \frac{1}{K-1}\left(1 - \frac{\left(X_{ijk} - \overline{X}_{ij}\right)^2}{\sum_{h=1}^{K}\left(X_{ijl} - \overline{X}_{ij}\right)^2}\right)$$

The weighted average for the parameter X is:

$$X_{ij} = \sum_{k=1}^{K} p_{ijk}^{(X)} X_{ijk}$$

Repeatability

- Repeatability indices for the parameters:

$$R_j^{\Delta} = \sum_{i=1}^{p}\sum_{k=1}^{K} \frac{\left|\Delta_{ij.} - \Delta_{ijk}\right|}{p * K * \Delta_{ij.}} \qquad R_j^{Imax} = \sum_{i=1}^{p}\sum_{k=1}^{K} \frac{\left|Imax_{ij.} - Imax_{ijk}\right|}{p * K * Imax_{ij.}}$$

- Repeatability indices for the skeleton:

$$R_j^{Sk} = \sum_{i=1}^{p}\sum_{k=1}^{K}\sum_{t=0}^{1} \frac{\left(Z_{ij.}(t) - Z_{ijk}(t)\right)^2}{p * K * 101}$$

⇨ Global repeatability indices: $R_j = R_j^{Sk} * R_j^{\Delta} * R_j^{Imax}$

$$0 \le R_j \le 1$$

Analysis of variance

For each assessor j and each parameters X (S, To, Δ), the ANOVA model is:

$$X_{ik}^{(j)} = \mu^{(j)} + \alpha_i^{(j)} + \varepsilon_{ik}^{(j)}$$

We are looking for the significance of the product effect

Fisher 's Discriminant Analysis

$$z_1 = Xa_1$$

$$\mathrm{a}_1 = \mathrm{Max}\frac{\mathrm{a}^{\mathrm{T}} B \mathrm{a}}{\mathrm{a}^{\mathrm{T}} T \mathrm{a}}$$

Between-class Variance

Total Variance

First Discriminant Axis